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THEORY FOR THE DESIGN OF UNDERGROUND PRESSURE CONDUITS

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POWER DIVISION

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THEORY FOR THE DESIGN OF UNDERGROUND PRESSURE CONDUITS

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SYNOPSIS

A theory for the design of underground pressure conduits is presented. Assuming that such conduits consist of steel liners concreted in shafts or tunnels excavated in sound rock, the theory for designing the steel liners is based on equating the deflections of steel and rock. There are so many uncertainties inherent in this problem that a scientifically exact solution is not possible. The theory is accordingly suggested as a rough and usable approach to a solution. It at least gives an indication of the order of magnitude of stresses respectively carried by the steel and the rock.

The effective modulus of elasticity of the rock must be known; this is not the modulus determined by laboratory tests of small samples of rock, but the modulus of the rock mass in place, and it can only be determined by field test. A suggested method of field testing is given, with an appropriate theory for application to test results to determine the modulus.

INTRODUCTION

Particularly in hydroelectric developments, underground pressure conduits are required. In recent years, quite a few underground powerhouses have been built, and for these, underground pressure conduits are a necessity. When powerhouses are built above ground, it is often advisable to place the pressure conduits underground, to avoid possible damage from snowslides, landslides, or for other reasons.

In the design of an underground pressure conduit, it would appear logical to take account of the fact that the rock mass must offer considerable resistance to bursting pressure, and that the steel liner therefore does not need to be as strong as it would have to be if in the open air and itself carrying the entire bursting pressure. If such account is not taken, the design is wasteful.

Naturally Existing Stresses In Rock

It is necessary to make assumptions as to compressive stresses naturally existing in the rock. The rock is therefore considered as a homogenous material. Of course, it is not, and the actual existing stresses will vary considerably from point to point. But on the average, it would appear that stresses must approximate those in a homogenous material. Symbols will be used as follows:

1. Cons. Engr., Sverdrup & Parcel, San Francisco, Calif.

σ_y = unit stress in vertical direction, in psi.
σ_x = unit stress in one horizontal direction, in psi.
σ_z = unit stress in second horizontal direction, in psi.
μ = Poisson's ratio for rock.
h = depth of rock cover over centerline of conduit, in feet. This is an approximation; strictly speaking, a value (h - a) should be used. A definition of "a" will appear later.
s = specific gravity of rock.
w = 62.4 pounds.

$$\sigma_z = \mu(\sigma_x + \sigma_y) \tag{1}$$

$$\sigma_x = \mu(\sigma_z + \sigma_y) \tag{2}$$

Eliminating σ_z by combining (1) and (2), we obtain

$$\sigma_x = \frac{\mu}{1 - \mu}\,\sigma_y \tag{3}$$

Since σ_x is unit stress in a horizontal direction, caused by the dead weight of rock and the Poisson effect, and since

$$\sigma_y = \frac{h\,s\,w}{144}$$

$$\sigma_x = \frac{\mu}{1 - \mu} \cdot \frac{s\,w}{144} \cdot h \tag{4}$$

Under our assumptions, σ_x is the minimum existing stress in the rock at depth "h"; $\sigma_z = \sigma_x$; in all other directions the stresses are greater.

Of course, μ and s must always be determined for the particular rock concerned. However, henceforth in this presentation it will be assumed that $\mu = 0.2$, and s = 2.65. Thus (4) becomes

$$\sigma_x = 0.287\ h \tag{4a}$$

Conditions In and Around Steel Liner Concreted in Rock

The water pressure within the liner will be resisted partly by the liner and partly by the rock, the proportion being resisted by each being determined by the condition that deflections are equal. That is, the radial deflection of the liner must equal the radial deflection of rock. Symbols will be used as follows:

P = water pressure in liner, in psi.
P_s = water pressure resisted by liner, in psi.
P_x = unit pressure in radial direction, carried by rock at distance "x" from centerline of liner, in psi.
f = unit circumferential stress in liner, in psi.
t = thickness of liner, in inches.
r = radius of liner, in inches.
E_s = 30,000,000, modulus of elasticity of steel.
μ_s = Poisson's ratio for steel, herein taken as 0.285
d_s = radial deflection of steel liner, in inches.

$$f = \frac{P_s r}{E_s t} \qquad \text{and} \qquad d_s = \frac{f r}{E_s}\ , \text{ whence}$$

$$d_s = \frac{P_s r^2}{E_s t} \tag{5}$$

But, since the liner is longitudinally restrained,

$$d_s = \frac{P_s r^2}{E_s t}(1 - \mu_s^2) \qquad (5a)$$

To determine deflections of the rock, we must theorize as to how the rock will act. The water pressure creates hoop tension in the liner, and tends to create hoop tension in the rock. Rock will act as though it were taking tension when the existing compressive stresses exceed the induced hoop tension; otherwise, because of its natural jointing, it is incapable of taking tension. The hoop tension induced in the rock, or tending to be induced, varies inversely as the distance from centerline of liner. It will therefore be greatest close to the liner, and will decrease as distance from the liner increases. Near the liner, it will probably exceed the existing compression in the rock, and the cracking in the rock will be accentuated. At some point farther from the liner, the existing compression will equal the hoop tension, and from this point on out the rock will act as a thick shell taking tension. Thus there will be a "cracked" zone around the liner, in which cracks will be produced in addition to the natural jointing, surrounded by a thick shell in which no new cracks will be produced but in which the existing compression will be reduced. In this presentation, the concrete around the steel will be neglected.

Proposed Theory

Referring to Figure 1, and using the following notation:

- a = radial distance to outer boundary of "cracked" zone (inner boundary of thick shell), in feet.
- b = radial distance to outer boundary of thick shell (rock surface), in feet.
- σ_a = unit hoop tension at a radial distance "a", psi.
- P_a = unit radial pressure at a radial distance "a", psi.
- E_r = effective modulus of elasticity of rock mass (not of an individual sample).
- d_o = radial deflection of a thick shell at its inner radius, in inches.
- d_i = radial deflection of "cracked" zone, in inches.
- d = total radial rock deflection, in inches.

the stress at the inner radius of a thick shell is:

$$\sigma_a = P_a \frac{b^2 + a^2}{b^2 - a^2} \qquad (6)$$

If $b > 20a$, the fraction $\frac{b^2 + a^2}{b^2 - a^2}$ is very close to unity, and can be considered as unity without appreciable error. Hence:

$$\sigma_a = P_a \qquad (6a)$$

The pressure P_a is related to the water pressure in the liner as follows:

$$P_a = (P - P_s)\frac{r}{12a} \qquad (7)$$

Rearranging:

$$12a = (P - P_s)\frac{r}{P_a} \qquad (7a)$$

and likewise

$$12a p_a = (P - P_s)r \tag{7b}$$

Assume that σ_a is the value of hoop tension directly above the liner at the outer boundary of the "cracked" zone, and set it equal to σ_x in (4a), thus:

$$\sigma_a = \sigma_x = 0.287\,h = P_a$$

It should really be

$$\sigma_a = \sigma_x = 0.287\,(h - a)$$

but "a" is usually small compared with "h"; the approximation is reasonable and simplifies matters. Thus P_a is a constant. Knowing P_a, and assuming a value for P_s, a corresponding value of "a" can be computed from (7a).

The radial deflection of a thick shell at its inner boundary is given by the formula

$$d_o = \frac{12a p_a}{E_r}\left(\frac{b^2 + a^2}{b^2 - a^2} + \mu\right) \tag{8}$$

The radial deflection of the "cracked" zone is now to be determined. Consider that there is no hoop tension, that the rock is divided into small columns, that one of the cross-sectional dimensions of a column is constant, and the other increasing with distance "x" from centerline of conduit. At this distance "x", expressed in inches, the unit pressure is, from (7):

$$P_x = (P - P_s)\frac{r}{x} \qquad \text{and}$$

$$d\,d_i = K\frac{P_x}{E_r}\,dx$$

K is a constant taking into account Poisson's ratio, and can be considered as unity: thus

$$d_i = (P - P_s)\frac{r}{E_r}\int_r^{12a}\frac{dx}{x}$$

$$d_i = (P - P_s)\frac{r}{E_r}\cdot \log_e x\Big]_r^{12a}$$

$$d_i = (P - P_s)\frac{r}{E_r}\log_e\frac{12a}{r} \tag{9}$$

Rewriting this to use common logarithms rather than natural:

$$d_i = 2.3(P - P_s)\frac{r}{E_r}\cdot \log\frac{12a}{r} \tag{10}$$

The total rock deflection is the sum of the deflections in the thick shell and the cracked zone, thus:

$$d = d_o + d_i \tag{11}$$

For rock and steel to be in equilibrium

$$d = d_o + d_i = d_s \tag{12}$$

Procedure in Using Theory

First Step. The value of P being known, assume values for P_s and t.

Second Step. Compute the radial deflection of the steel liner, d_s, from (5a).

Third Step. Compute $P_a = 0.287h$. Compute 12a and $12aP_a$ from (7a) and (7b). Compute rock deflections, d_o and d_i, from (8) and (9). Compute d from (11).

Fourth Step. See if the values for d and d_s satisfy (12). If they do not, make a new assumption for either P_s or t, or new assumptions for both. Repeat until (12) is satisfied. The thickness t of the steel shell will then have been determined.

Illustrative Example

It is desired to place a 75-inch diameter steel penstock underground. We have given:

$P = 740$ psi $\qquad E_r = 2{,}500{,}000$

$h = 500'$ $\qquad r = 37.5''$

$b = 500'$

Try a thickness of one-half inch for the steel. Assume that

$$P_s = 225 \text{ psi.}$$

Compute the deflection of the steel, thus:

$$d_s = \frac{P_s r^2}{E_s t}(1 - \mu_s^2) \qquad 1 - \mu_s^2 = 0.9188 \qquad (5a)$$

$$\frac{r^2(1 - \mu_s^2)}{E_s} = \frac{1406.25 \cdot 0.9188}{3 \cdot 10^7} = \frac{430.69}{10^7}$$

$$d_s = \frac{430.69}{10^7} \cdot 225 \cdot 2 = 0.01940''$$

$$12aP_a = (P - P_s)\,r \qquad (7)$$

$$= (740 - 225)\,37.5 = 19{,}312.5$$

$$12a = \frac{19{,}312.5}{0.287 \cdot 500} = 134.6$$

$$d_o = \frac{12aP_a}{E_r}\left(\frac{b^2 + a^2}{b^2 - a^2} + \mu\right) \qquad (8)$$

Since b is very large compared to a, the fraction $\frac{b^2 + a^2}{b^2 - a^2}$ will be considered as unity.

$$d_o = \frac{19{,}312.5}{2.5 \cdot 10^6}(1 + 0.2) = 0.00928''$$

$$d_i = 2.3\,(P - P_s)\frac{r}{E_r} \cdot \log \frac{12a}{r} \qquad (10)$$

$$\frac{2.3 \cdot 37.5}{2.5 \cdot 10^6} = \frac{34.5}{10^6}$$

$$d_i = \frac{34.5}{10^6} \cdot 515 \cdot \log \frac{134.6}{37.5} = 0.00988''$$

$$d = d_o + d_i = 0.00928 + 0.00988 = 0.01916''$$

The value of d is slightly smaller than the value of d_s, and therefore the trial value of 225 assumed for P_s is a little too large. By inspection, P_s should be about 223 psi. Thus the steel carries 223 psi out of the total bursting pressure of 740 psi, and the rock carries 517. The stress in the steel is

$$223 \cdot 37.5 \cdot 2 = 16{,}725 \text{ psi.}$$

Table I is introduced to show how the distribution of load between steel and rock varies with thickness of steel and modulus of elasticity of rock.

Determination Of Effective Modulus Of Elasticity Of Rock

On one large project, the effective modulus of elasticity of rock in the field was determined by the writer as follows: a sphere of one-half inch steel plate, 10 feet in diameter, was concreted in the rock, and placed under varying water pressure, up to 3,600 psi. A sphere was used instead of a cylinder, because a cylinder would necessarily have been short, and the end conditions would have made evaluation of results very difficult. A theory for the sphere is presented in the following, without all the explanation before given for the cylinder. As before:

$$\sigma_x = \frac{\mu}{1 - \mu} \sigma_y \qquad (3)$$

The radial deflection, d_s, of the steel sphere is obtained as follows:

$$f = \frac{P_s r}{2t}$$

and

$$d_s = \frac{fr}{E_s} (1 - \mu_s)$$

whence

$$d_s = \frac{P_s r^2}{2 E_s t} (1 - \mu_s) \qquad (13)$$

The circumferential unit stress at the inner radius of a spherical thick shell is

$$\sigma_a = P_a \frac{b^3 + 2a^3}{2(b^3 - a^3)} \qquad (14)$$

For any value of b in excess of 10a, to all intents and purposes

$$\sigma_a = \frac{P_a}{2} \qquad (14a)$$

The unit radial pressure at a distance a from the center of the sphere is related to the water pressure in the sphere thus:

$$P_a = (P - P_S)\frac{r^2}{144a^2} \tag{15}$$

At a distance a above the center of the sphere

$$\sigma_a = \sigma_x = 0.287(h - a)$$

Hence

$$P_a = 0.574(h - a) \tag{16}$$

Combining (15) and (16)

$$0.574(h - a) = (P - P_S)\frac{r^2}{144a^2}$$

Rearranging:

$$a^3 - a^2h + \frac{r^2}{82.656}(P - P_S) = 0 \tag{17}$$

If P_S is assumed, a corresponding value of a can be determined by trial. However, a is usually small compared to h, and little error is involved in using h instead of (h - a) in (16), giving

$$a^2 = \frac{r^2}{82.656h}(P - P_S) \tag{17a}$$

It is to be borne in mind that only the small positive root of (17) has any significance.

The radial deflection of a thick spherical shell at its inner radius is

$$d_o = \frac{12aP_a}{E_r}\left[\frac{b^3 + 2a^3}{2(b^3 - a^3)} \cdot (1 - \mu) + \mu\right], \text{ but, to all intents and purposes } \frac{b^3 + 2a^3}{b^3 - a^3} = 1, \text{ so}$$

$$d_o = \frac{12aP_a}{E_r}\left[\frac{1 - \mu}{2} + \mu\right] \tag{18}$$

$$= \frac{6aP_a}{E_r}(1 + \mu) \quad \text{and if } \mu = 0.2$$

$$d_o = \frac{7.2aP_a}{E_r} \tag{18a}$$

Within the "cracked" zone, the radial deflection of any element dx is

$$d\,d_i = \frac{P_x}{E_r}\,dx$$

Substituting the value for P_x given by (15)

$$d\,d_i = (P - P_S)\frac{r^2}{E_r}\frac{dx}{x^2} \quad \text{where r and x are both expressed in inches.}$$

$$d_i = -(P - P_S)\frac{r^2}{E_r} \cdot \frac{1}{x}\Bigg]_r^{12a}$$

$$d_i = (P - P_S)\frac{r}{E_r}\left(\frac{12a - r}{12a}\right) \tag{19}$$

Now, $d = d_s = d_o + d_i$ must be satisfied.

Assuming that a steel sphere has been installed and tested, and the radial deflections of the sphere have been measured;

First Step. Substitute the ascertained value of d_s in (13), and compute P_s.

Second Step. Substitute the values of P and P_s in (17a), and compute a.

Third Step. Substitute the values of a and P_s in (15), and compute P_a.

Fourth Step. Adding (18a) and (19), we get

$$d = d_s = \frac{7.2aP_a}{E_r} + \frac{r}{E_r}(P - P_s)\left(\frac{12a - r}{12a}\right) \qquad \text{whence}$$

$$E_r = \frac{1}{d_s}\left[7.2aP_a + r(P - P_s)\left(\frac{12a - r}{12a}\right)\right] \qquad (20)$$

and E_r is thus determined.

The use of a steel sphere is expensive, and can be justified only for a big development. It is hoped that readers can suggest other cheaper ways of determining E_r, bearing in mind that it is a property of the rock mass in place.

CONCLUSION

It is obvious that there are many technical objections to this theory. The ratio between horizontal and vertical stresses naturally existing in rock may be quite different from that assumed. No correction has been made for the fact that the depth of rock should be reduced by the outer radius of the "cracked" zone. Concrete around the liner has been neglected. This concrete may be so placed that it does not transmit pressure uniformly in all radial directions.

But, at present, to the writer's knowledge, no one has previously made any theoretical approach at all to the determination of the proportioning of load between steel and rock. Steel liners have been designed by guess. In fairly good and uniform rock there seems to be no good reason why the load-carrying capacity of the rock should not be considered in designing the liner. There are, of course, other considerations than internal pressure: the steel conduit must be deep enough so that sheer weight of rock is greater than the bursting pressure, and it must be strong enough so it will not buckle under possible external pressure caused by ground water when it is drained. No really good way of avoiding such possible external pressure has as yet been developed.

The writer suggests that the actual effective modulus of elasticity of the rock be determined and that a value of about two-thirds of this be used in design. He believes that the use of this theory will then result in sound and economical design. It is his hope that the presentation of this theory may bring out discussion which will perhaps lead to a better understanding of the problem of designing underground pressure conduits.

TABLE I

VARIATION OF PRESSURE CARRIED BY UNDERGROUND PRESSURE CONDUIT

AS STEEL THICKNESS VARIES

AS MODULUS OF ELASTICITY OF ROCK VARIES

P = 740 psi
h = 500'
b = 500
r = 37.5"

E_r	t in.	P_s psi	$12aP_a$	12a	d_o in.	d_i in.	d in.	d_s in.	f psi
$2.5 \cdot 10^6$	1/4	138	22,575	157.18	.01084	.01293	.02377	.02377	20,700
	1/2	223	19,387.5	135.10	.00931	.00993	.01924	.01920	16,725
	3/4	282	17,175	119.6	.00825	.00797	.01622	.01620	14,100
	1	326	15,525	108.19	.00745	.00657	.01402	.01404	12,225
$1.5 \cdot 10^6$	1/4	198	20,325	141.64	.01626	.01799	.03425	.03411	29,700
	1/2	298	16,575	115.48	.01326	.01239	.02565	.02567	22,350
	3/4	360	14,250	99.30	.01140	.00924	.02064	.02067	18,000
	1	402	12,675	88.33	.01014	.00723	.01737	.01731	15,075

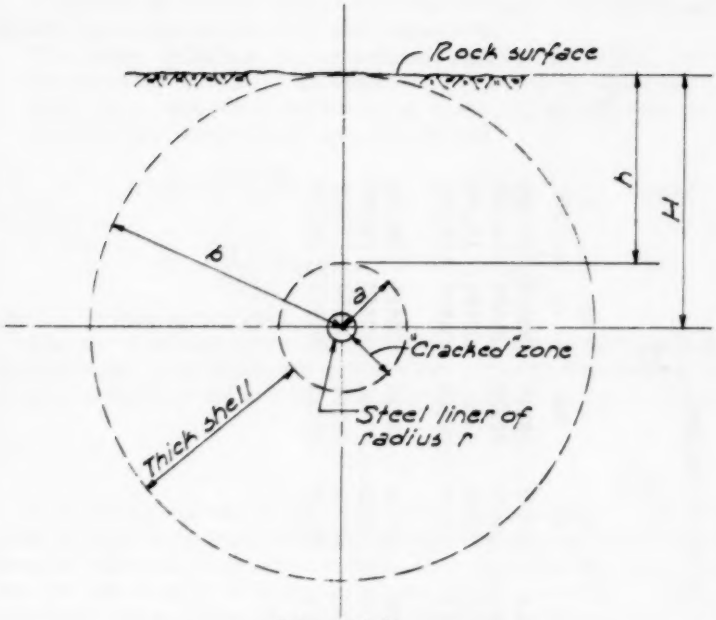

Fig. 1(a)
Cross-Section Of Underground Pressure Conduit

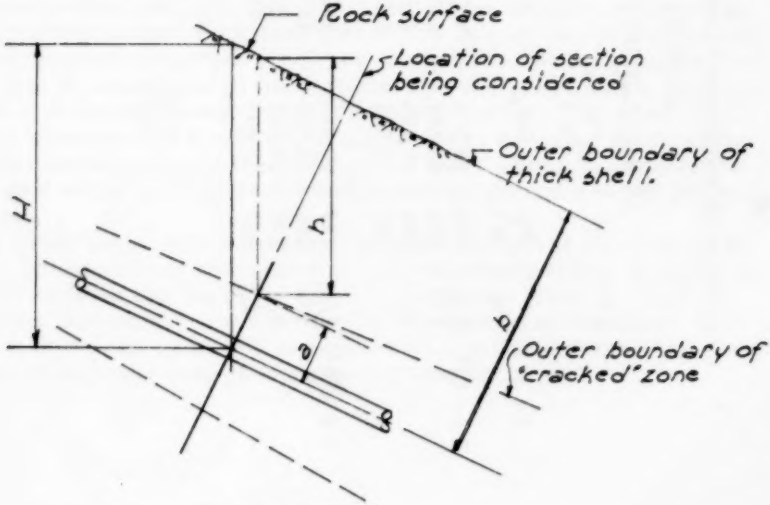

Fig. 1(b)
Longitudinal Section Of Underground Pressure Conduit